Math 240 Spring 2015 -- More Practice Problems for Exam 1

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1. Find *all* solutions of the following equations - or show that there is none:

   a)

$$x_1 + x_2 + x_3 - 2x_4 = 0$$
$$x_1 + x_2 + 3x_3 - 2x_4 = 0$$

   b)

$$x_1 + x_2 = 1$$
$$x_1 - x_2 = 3$$
$$2x_1 + x_2 = 3$$

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2. Determine whether the matrix

$$A = \begin{pmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \\ 2 & 3 & 5 \end{pmatrix}$$

   is invertible. Find its inverse if it is.

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3. Let A and B be 2-by-2 matrices. We say that A and B commute, if $AB = BA$. Show that if A and B both commute with

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

   then A commutes with B also.

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4. Use Gaussian elimination to find the values of b for which the following linear system has a solution. Find the corresponding solution(s).

$$x_2 + 2x_3 = 4$$
$$x_1 + 2x_2 + 5x_3 = 6$$
$$-x_2 - 2x_3 = b$$

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5. Compute the determinant of

$$\begin{pmatrix} 1 & 6 & 11 & 16 & 21 \\ 2 & 7 & 12 & 17 & 22 \\ 3 & 8 & 13 & 18 & 23 \\ 4 & 9 & 14 & 19 & 24 \\ 5 & 10 & 15 & 20 & 25 \end{pmatrix}.$$

Hint: Very little computing is needed.

True/False Problems. For each of the following statements determine whether the statement is true or false. Explain your answer.

1. Let A be a square matrix. If $A^3 = 0$, then $\det(A) = 0$.
2. Let A, B and C be n-by-n matrices. If $AB=AC$ and A is invertible, then $B=C$.
3. Let L be an invertible map from the plane R^2 to itself has the property that it is its own inverse, $L = L^{-1}$, then $L = \pm I$ where I is the identity map.
4. If A and B are n-by-n matrices with A invertible, then $(ABA^{-1})^2 = AB^2A^{-1}$.
5. Say A is a 4-by-4 matrix for which $\det(A) = -3$. Then $\det(2A) = -6$.
6. Let A, B be two real 5-by-5 matrices. Then $\det(A + B) = \det(A) + \det(B)$.
7. If the 3-by-3 matrices A, B are both nonsingular, then $A+B$ is also nonsingular.
8. If the 4-by-4 matrices A, B are both symmetric, then $A+B$ is also symmetric.
9. Let **a**, **b**, **c** and **d** be *non-zero* vectors in R^3. It is impossible that **a**, **b**, **c** and **d** are linearly independent.

For the following three questions, consider a system of linear algebraic equations written in matrix form

$$A \cdot \mathbf{x} = \mathbf{b} \ ,$$

where A is an n-by-n matrix with $\det(A)=0$,

$$\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \in \mathbf{R}^n, \quad \text{and} \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \ ,$$

$x_1,\dots,x_n$ are the unknowns.

10. It is impossible that for some vector **b** there is exactly one solution.
11. If $\mathbf{b} = \mathbf{0}$, then there are infinitely many solutions.
12. For all vectors **b** there is at least one solution.