MATH 240 – Practice problems for First Midterm Exam - Spring 2015

1. The determinant of the matrix $\begin{bmatrix} 2 & 0 & 1 & 0 \\ 2 & 3 & 3 & 1 \\ -3 & 2 & 1 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ is -6. What is the determinant of the matrix $\begin{bmatrix} -3 & 2 & 1 & 2 \\ 4 & 6 & 6 & 2 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}$?

of the matrix $\begin{bmatrix} 4 & 3 & 4 & 1 \\ 2 & 3 & 3 & 1 \\ 0 & 1 & 2 & 1 \\ -3 & 2 & 1 & 2 \end{bmatrix}$?

of the matrix $\begin{bmatrix} -2 & -3 & -3 & -1 \\ 3 & -2 & -1 & -2 \\ 0 & -1 & -2 & -1 \\ -2 & 0 & -1 & 0 \end{bmatrix}$?

of the matrix $\begin{bmatrix} 4 & 0 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 2 & 3 & 3 & 1 \\ -3 & 2 & 1 & 2 \end{bmatrix}$?

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2. For each of the following subsets of the vector space P_4 of polynomials of degree less than or equal to 4, say whether or not it is a (vector) subspace of P_4. If it is not a subspace, explain why not. If it is a subspace, give its dimension and a basis for the subspace.

(a) The set of polynomials in P_4 that are even functions (i.e., for which $p(-x) = p(x)$.

(b) The set of polynomials in P_4 that are odd functions (i.e., for which $p(-x) = -p(x)$.

(c) The set of polynomials in P_4 that satisfy $p(0) = 1$ and $p(1) = 2$.

(d) The set of polynomials in P_4 that satisfy $p(0) = 0$ and $p(1) = 0$.

(e) The set of polynomials in P_4 that satisfy $p(1) = 0$, $p'(1) = 0$ and $p''(1) = 0$.

(f) The set of polynomials in P_4 that satisfy $p(1) = 1$ and $p'(1) = 2$.

3. Consider the matrix $A(k) = \begin{bmatrix} 1 & 1 & -2 \\ 1 & k & 0 \\ -1 & 2 & k \end{bmatrix}$.

(a) There are two values of k for which the rank of the matrix $A(k)$ is less than three. What are they?

(b) For each of those values of k, find a basis for the nullspace of $A(k)$.

(c) For one of the values of k, it is possible to solve $A(k)\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{b} = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}.$$

What is the general solution of this problem for this value of k?

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4. For the matrix

$$M = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

determine the dimension of the subspace of 3-by-3 matrices X for which

$$MX = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Also give a basis for this subspace.

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5. Let $S = \{1, x, x^2\}$ be the standard basis for the vector space P_2 of polynomials of degree less than or equal to 2.

(a) Show that $B = \{1 + x, 1 + x^2, x + x^2\}$ is another basis for P_2.

(b) What is the change-of-basis matrix $P_{S \leftarrow B}$ (in other words how do you go from expressing a polynomial as $a_1(1 + x) + a_2(1 + x^2) + a_3(x + x^2)$ to expressing it as $b_1(1) + b_2(x) + b_3(x^2)$)?

(c) What is the change-of-basis matrix $P_{B \leftarrow S}$?

(d) What is the matrix the represents the linear mapping that sends $p(x)$ to $p'(x) + 2p(x)$ with respect to the standard basis S?

(e) What is the matrix that represents the linear mapping in part (d) with respect to the basis B?

6. (a) Can the vector $\begin{bmatrix} 2 \\ 1 \\ 5 \\ 0 \end{bmatrix}$ be represented as a linear combination of the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 6 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$? If not, explain why not. If so, how? (Be precise – if there is more than one way to do it, give all possible ways).

(b) Same question, but for the vector $\begin{bmatrix} 2 \\ 1 \\ 4 \\ 1 \end{bmatrix}$

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7. Let $A = \begin{bmatrix} 1 & 0 & -1 & -2 & 0 & 0 \\ -2 & -1 & 0 & 2 & 0 & -1 \end{bmatrix}$.

(a) Explain why the matrix $A^T(AA^T)^{-1}$ would be a right inverse for A, provided it exists.

(b) Calculate $A^T(AA^T)^{-1}$ and show that it is a right inverse for A. (Sorry about the fractions!)