Homework 1

DUE: THURSDAY, JANUARY 23

This week. Read 2.3-2.5 in the book.

1. In the class, we discussed the following system of equations related to Google PageRank.

$$
\begin{aligned}
x_1 &= \frac{1}{2}x_2 + x_4,\\
x_2 &= \frac{1}{3}x_1 + \frac{1}{2}x_3,\\
x_3 &= \frac{1}{3}x_1,\\
x_4 &= \frac{1}{3}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3.
\end{aligned}
$$

   Use row reduction to find the following solution

$$
\begin{aligned}
x_1 &= \frac{4}{3}x_4,\\
x_2 &= \frac{2}{3}x_4,\\
x_3 &= \frac{4}{9}x_4.
\end{aligned}
$$

2. Consider vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

   a) Draw a picture to explain why there are no numbers $c_1, c_2 \in \mathbb{R}$ such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 = \mathbf{b}$.

   b) Reinterpret the equation $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 = \mathbf{b}$ as a system $A\mathbf{x} = \mathbf{b}$ of three equations in two variables (c_1 and c_2) and reinterpret your answer in this notation. What are A and $\mathbf{x}$?

3. Consider the vectors $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$.

   a) Using the cross product, find a normal vector to the plane containing $\mathbf{v}$ and $\mathbf{w}$.

   b) By solving a system of equations, find a normal vector to the plane containing $\mathbf{v}$ and $\mathbf{w}$. [Hint: recall that two vectors are orthogonal precisely when their dot product is zero.]

4. Consider the vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ in $\mathbb{R}^4$. Solve a system of equations to find a normal vector to the hyperplane in $\mathbb{R}^4$ containing $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$. Note: there is no good generalization of the cross product to higher dimensions.