Homework 10

DUE: SUNDAY, APRIL 19

This week. Read 8.1-8.5

Spring-Mass system

1. True/False
   a) An underdamped spring-mass system tends to rest as $t \to \infty$.
   b) Underdamped, critically damped, and overdamped spring-mass systems exhibit periodic motion.
   c) The larger the mass, the shorter the period of a spring mass system that is undergoing simple harmonic motion.
   d) The initial conditions (initial position and velocity) can change whether a spring-mass system is underdamped, critically damped, or overdamped.

2. Determine the motion of the spring-mass system governed by the initial value problem. In each case, state whether the motion is underdamped, critically damped, or overdamped.
   a) $y'' + 4y' + 7y = 0, y(0) = 2, y'(0) = 6.$
   b) $y'' + 3y' + 2y = 0, y(0) = 1, y'(0) = 0.$

3. Consider the spring-mass system whose motion is governed by the initial-value problem

$$y'' + \frac{1}{5}y' + \frac{1}{100}y = 0, \quad y(0) = 1, \quad y'(0) = 5.$$

   Determine the position of the mass at time t.

4. Consider the spring-mass system whose motion is governed by the initial-value problem

$$y'' + 2y' + 5y = 17 \sin 2t, \quad y(0) = -2, \quad y'(0) = 0.$$

   a) Determine whether the motion is underdamped, overdamped, or critically damped.
   b) Find the solution to the given initial-value problem and identify the steady-state and transient parts.

Reduction of order

5. In the problems below, y_1 is a solution to the given differential equation. Use the method of reduction of order to determine a second linearly independent solution.

a) $x^2y'' - 2xy' + (x^2 + 2)y = 0, x > 0, y_1(x) = x \sin x.$

b) $(1 - x^2)y'' - 2xy' + 2y = 0, -1 < x < 1, y_1(x) = x.$

6. Determine the general solution to the given differential equation.

a) $x^2y'' - 3xy' + 4y = 8x^4, x > 0.$

b) $y'' - 4y' + 4y = 4e^{2x} \ln x, x > 0.$