Homework 8

DUE: FRIDAY, APRIL 5TH

This week. Read 8.1-8.2.

1. Use the ideas in diagonalization to solve the system of differential equations of $x(t), y(t)$

$$x' = x + 4y$$
$$y' = 2x + 3y.$$

2. Compute the matrix exponential e^{At} for the following matrices A.

   a) $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

   b) $A = \begin{bmatrix} 6 & -2 & -1 \\ 8 & -2 & -2 \\ 4 & -2 & 1 \end{bmatrix}$ (You may use that $p(\lambda) = -(\lambda - 2)^2(\lambda - 1)$.)

3. True/False

   a) If $y_1, y_2, \ldots, y_n$ are solutions to a regular nth order linear homogeneous differential equation such that $W[y_1, y_2, \ldots, y_n](x)$ is zero at exactly n points of I, then $\{y_1, \ldots, y_n\}$ is a linearly independent set of functions.

   b) If L_1 and L_2 are linear differential operators, then $L_1 L_2 = L_2 L_1$.

   c) If y_p is a particular solution to the differential equation $Ly = F$, then $y_p + u$ is also a solution to $Ly = F$ for every solution u of the corresponding homogeneous differential equation $Ly = 0$.

4. Verify by direct substitution that xe^x is in the kernel of $D^2 - 2D + 1 = (D - 1)^2$.

5. Compute the kernel of L, where $L = x^2 D + x$.

6. Determine which of the following sets of vectors is a basis for the solution space to the differential equation $y'' - 16y = 0$:

$$S_1 = \{e^{4x}\}, \quad S_2 = \{e^{2x}, e^{4x}, e^{-4x}\}, \quad S_3 = \{e^{4x}, e^{2x}\},$$
$$S_4 = \{e^{4x}, e^{-4x}\}, \quad S_5 = \{e^{4x}, 7e^{4x}\}, \quad S_6 = \{\cosh 4x, \sinh 4x\}.$$

7. Goode and Annin: 8.1.25, $y'' - 36y = 0$ (Look at the hint in the textbook $y = e^{rx}$)

8. Goode and Annin: 8.1.36, $x^3 y''' + x^2 y'' - 2xy' + 2y = 0, x > 0$ (Look at the hint in the textbook $y = x^r$)