Homework 9

Due: Thursday, April 9

This week. Read 8.1-8.3.

1. This problem justifies a fact we have used in class: that $\{e^{\lambda t}, te^{\lambda t}, t^2e^{\lambda t}, \ldots, t^{m-1}e^{\lambda t}\}$ is a basis for the kernel of the operator $(D-\lambda)^m$.
   a) If $p(t)$ is a polynomial and λ is a scalar, show that
   $$(D-\lambda)\left(p(t)e^{\lambda t}\right) = p'(t)e^{\lambda t}.$$
   b) If $p(t)$ is a polynomial of degree less than m, what is
   $$(D-\lambda)^m\left(p(t)e^{\lambda t}\right)?$$
   c) Find a basis of the kernel of the linear differential operator $(D-\lambda)^m$.

2. What is the dimension n of the space of solutions to the linear ODE $u''' + 9u' = 0$? Find n functions that give a basis of this vector space, and use them to write a formula for the general solution $u(t)$.

3. Determine the general solutions of the following equations:
   a) $y'' - y' - 2y = 0$.
   b) $y'' - 6y' + 9y = 0$.
   c) $(D+2)^2y = 0$.
   d) $y''' - y'' + y' - y = 0$.
   e) $y''' - 2y'' - 4y' + 8y = 0$.

4. Determine the annihilator of the given functions:
   a) $F(x) = x^3e^{7x} + 5\sin 4x$.
   b) $F(x) = (1-3x)e^{4x} + 2x^2$.

5. Find the general solution to the following differential equation. Derive your trial solution using the annihilator technique.
   a) $(D^2+16)y = 4\cos x$
   b) $(D+1)(D-3)y = 4(e^{-x} - 2\cos x)$.