1. Pf: ① $1 \in H$, $1 \in K$. $1 \in H \cap K$ (Identity)

② $a \in H \cap K$, $b \in H \cap K$. then $ab \in H$, $ab \in K$.

so $ab \in H \cap K$ (closure)

③ $a \in H \Rightarrow a^{-1} \in H$. $a \in K \Rightarrow a^{-1} \in K$.

so $a^{-1} \in H \cap K$. (Inverse).

2. $S = \{ \text{identity}, (12) \}$.

choose $g = (23)$.

then $g(12)g^{-1} = (13) \notin S$

3. let $\varphi : x \mapsto \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$

then $\varphi(x+y) = \begin{pmatrix} 1 & x+y \\ & 1 \end{pmatrix}$

$\varphi(x)\varphi(y) = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & x+y \\ & 1 \end{pmatrix} = \varphi(x+y)$

4. ① Transitive. $a \sim b$. $b \sim c \Rightarrow b = g_1 a g_1^{-1}$, $c = g_2 b g_2^{-1}$

$\Rightarrow c = g_2 g_1 a g_1^{-1} g_2^{-1} = (g_2 g_1) a (g_2 g_1)^{-1} \Rightarrow a \sim c$

② Symmetric $a \sim b \Rightarrow b = gag^{-1} \Rightarrow a = (g^{-1}) b (g^{-1})^{-1}$

③ reflexive $a = 1a1^{-1}$