Math 371 Homework#11

Due on 4/25 at the beginning of Lecture

1. Prove that the polynomial $x^4 + 3x + 3$ is irreducible polynomial over the field $\mathbb{Q}[\sqrt[3]{2}]$. (Hint: Use multiplicative property of degree. In class, we proved $x^3 - 2$ is irreducible in $\mathbb{Q}[\sqrt{2}]$. Similar argument also works here.)

2. Determine the irreducible polynomial for $\alpha = \sqrt{3} + \sqrt{5}$ over the following fields. (You need to prove why they are the irreducible polynomials)

   (a) $\mathbb{Q}$

   (b) $\mathbb{Q}(\sqrt{15})$

   (c) $\mathbb{Q}(\sqrt[3]{2})$

   (Hint: try to use $\pm\sqrt{3} \pm \sqrt{5}$ as the roots of the polynomial to find some polynomial over $\mathbb{Q}$. Use the tower of the field extension to find $[\mathbb{Q}(\sqrt{3}, \sqrt{5}) \colon \mathbb{Q}(\sqrt{15})]$ and $[\mathbb{Q}(\sqrt{3} + \sqrt{5}, \sqrt[3]{2}) \colon \mathbb{Q}(\sqrt[3]{2})]$.

3. Prove that any quadratic extension of $\mathbb{R}$ is isomorphic to $\mathbb{C}$.

4. Prove that the characteristic of a field F is either 0 or a prime number. If it is a prime number p, show that the map $\phi \colon x \mapsto x^p$ gives an injective ring homomorphism from F to itself.