Math 371 Homework#4

Due on 2/14 at the beginning of Lecture

1. Find the center of the dihedral group D_4.
2. Classify all the finite groups G of order $|G| = 99$.
3. Consider the symmetric group S_5. Find the centralizer $Z(x)$ of the element $x = (123)$ and the normalizer $N(H)$ of the subgroup $H =< x >$ generated by x.
4. Find all the Sylow subgroups of S_4.
5. **Artin, Chapter 7, 3.2**
   Let Z be the center of a group G. Prove that if G/Z is a cyclic group, then G is abelian, and therefore $Z = G$.
6. **Artin, Chapter 7, 7.4(a)**
   A group G is *simple* if it is not the trivial group $\{1\}$ and if it contains no normal subgroup other than $\{1\}$ and G. Prove that no simple group has order pq, where p and q are prime.
7. **Artin, Chapter 7, 7.3**
   How many elements of order 5 might be contained in a group of order 20.