

## Math 371 Homework#5

Due on 2/21 at the beginning of Lecture

1. Prove the semidirect product  $H \rtimes_{\phi} K$  defined in the class gives a group structure on  $H \times K$ . Let  $H$  and  $K$  be two groups and  $\phi: K \rightarrow \text{Aut}(H)$  be a group homomorphism. Define a binary operation on  $H \times K$  by  $(h, k)(h', k') = (h\phi(k)(h'), kk')$ . Check this binary operation gives a group structure.
2. **Artin, Chapter 7, 8.6**  
Let  $G$  be a group of order 55
  - (a) Prove that  $G$  is generated by elements  $x$  and  $y$  with relations  $x^{11} = 1, y^5 = 1$  and  $xyx^{-1} = x^r$ .
  - (b) Find the possible values of  $r$ .
  - (c) Prove there are two isomorphism classes of  $G$ .