Math 371 Homework#6

Due on 2/28 at the beginning of Lecture

1. **Artin, Chapter 11, 1.8**
   Determine the units in $\mathbb{Z}/12\mathbb{Z}$, $\mathbb{Z}/8\mathbb{Z}$. (Hint: in the class, we used the fact that any element a which is not a zero divisor defines an injective map from R to R itself by multiplication by a. Since R is finite, this also defines an surjective map and hence 1 has a preimage. This proves that any non zero-divisor is a unit)

2. **Artin, Chapter 11, 1.7 (a)**
   Let U be an arbitrary set and R be the set of subsets in U. Addition and multiplication of elements of R are defined by $A + B = A \cup B - A \cap B$ and $A \cdot B = A \cap B$. Prove that R is a ring.

3. Determine whether the division with reminder $g(x) = f(x)q(x) + r(x)$ exists in $R[x]$ for the following $R, f(x), g(x)$. If it exists, find the $q(x), r(x)$.

   (a) $R = \mathbb{Z}, f(x) = 2x^2 + x + 1, g(x) = 2x^3 + 7x^2 + 4x + 8$

   (b) $R = \mathbb{Z}/6\mathbb{Z}, f(x) = 2x + 1, g(x) = 2x^2 + 2x$

   (c) $R = \mathbb{Z}/6\mathbb{Z}, f(x) = 5x + 1, g(x) = 2x^2 + 2x$