# Math 371 Homework#7

Due on 3/14 at the beginning of Lecture

1. Let $R$ be a ring and $I$, $J$ ideals of $R$. Prove the following
   (a) $I \cap J$ is an ideal of $R$,
   (b) $I + J = \{a + b | a \in I, b \in J\}$ is an ideal of $R$,
   (c) $IJ = \{\sum_{i=0}^{n} a_i b_i | a_i \in I, b_i \in J, n \in \mathbb{N}\}$ is an ideal of $R$.
2. Let $I = (a)$ and $J = (b)$ be two ideals of $R$. Prove that $I \subset J$ if and only if $b$ divides $a$. Use this fact and correspondence theorem to classify all the ideals in $\mathbb{Z}/12\mathbb{Z}$.
3. **Artin Chapter 11, 3.4** Let $\phi\colon \mathbb{C}[x, y] \to \mathbb{C}[t]$ defined by $x \mapsto t + 1$ and $y \mapsto t^3 - 1$. Determine the kernel $K$ of $\phi$ and prove that every ideal $I$ of $\mathbb{C}[x, y]$ that contains $K$ can be generated by two elements.
4. **Artin Chapter 11, 3.2** Prove that any ideal of Gaussian integers $\mathbb{Z}[i]$ must contain an integer.
5. **Artin Chapter 11, 4.3 a) b)** Identify the following rings
   (a) $\mathbb{Z}[x]/(x^2 - 3, 2x + 4)$ (NO solution in terms of $\mathbb{Z}/n\mathbb{Z}$ for this one, please discard this question),
   (b) $\mathbb{Z}[i]/(2 + i)$.

   with $\mathbb{Z}/n\mathbb{Z}$ for some $n$.
6. **Artin Chapter 11, 3.3 a) b)** Find generators of the kernels of the following maps:
   (a) $\mathbb{R}[x, y] \to \mathbb{R}$ by $f(x, y) \mapsto f(0, 0)$,
   (b) $\mathbb{R}[x] \to \mathbb{C}$ by $f(x) \mapsto f(2 + i)$.