Math 371 Homework#9

Due on 3/28 at the beginning of Lecture

1. **Artin, Chapter 11, 8.3** Prove that the ring $\mathbb{F}_2[x]/(x^3+x+1)$ is a field, but that $\mathbb{F}_3[x]/(x^3+x+1)$ is not a field.

2. Use the Euclidean domain structure described in Proposition 12.2.5 to divide -4 by $2+i$ in $\mathbb{Z}[i]$, i.e. find $q, r \in \mathbb{Z}[i]$ such that $-4 = (2+i)q+r$ and $r = 0$ or $\sigma(r) < \sigma(2+i)$. (Hint: use the picture in Proposition 12.2.5)

3. Assume a and b are associates in integral domain R. Prove that if a is irreducible, then b is also irreducible.

4. Prove that $\mathbb{C}[x,y]$ is not a PID (principal ideal domain). (Hint: consider the ideal (x,y) and prove that it can not be generated by one element. Assume it is generated by one element $f(x,y)$, then try to find the degree of $f(x,y)$ with respect to x (and y).)