

Math 371
Spring 2019
Midterm 1
02/19/2019
Time Limit: 80 Minutes

Name: _____

ID _____

“My signature below certifies that I have complied with the University of Pennsylvania’s Code of Academic Integrity in completing this”

Signature _____

This exam contains 12 pages (including this cover page) and 11 questions.
Total of points is 110.

- Check your exam to make sure all 12 pages are present.
- You may use writing implements and a single handwritten sheet of 8.5”x11” paper.
- NO CALCULATORS.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Good luck!

Grade Table (for teacher use only)

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
Total:	110	

1. (10 points) Let H be a subgroup of G . State the definition of normalizer of H in G . Find the normalizer of $H = \{1, (123), (132)\}$ in S_4 .

2. (10 points) Write the element $(123)(2345) \in S_5$ as product of disjoint cycles.

3. (10 points) Find the Sylow 2-subgroups of D_6 .

4. (10 points) Find all the normal subgroups of S_4 .

5. (10 points) Let

$$H = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$$

be a subgroup of special orthogonal group $SO(2)$. Prove that the quotient group $SO(2)/H$ is isomorphic to $SO(2)$.

6. (10 points) Let $Z(G)$ be the center of G . Prove that $|G/Z(G)|$ can not be 15.

7. (10 points) Prove that a group of order 56 is not a simple group.

8. (10 points) Classify all finite groups of order 14.

9. (10 points) Is there a transitive operation of S_4 on a set of five elements? Why?

10. (10 points) Classify finite groups of order 28.

Bonus Question

11. (10 points) Let p be a prime number and G be a p -group. Let H be a proper subgroup of G (a subgroup of G which is not equal to G). Prove that the normalizer $N(H)$ is strictly larger than H . (Hint: Restrict the operation of G on the cosets G/H to H).