

Math 371
Spring 2019
Midterm 2
4/2/2019

Name: _____

Time Limit: 80 Minutes

ID _____

“My signature below certifies that I have complied with the University of Pennsylvania’s Code of Academic Integrity in completing this”

Signature _____

This exam contains 10 pages (including this cover page) and 9 questions.
Total of points is 108.

- Check your exam to make sure all 10 pages are present.
- You may use writing implements on both sides of a sheet of 8.5”x11” paper.
- NO CALCULATORS.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Good luck!

Grade Table (for teacher use only)

Question	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
Total:	108	

1. (12 points) State the definition of principal ideal domain (PID). Give three examples of PID.

2. (12 points) Find the units in $\mathbb{Z}/6\mathbb{Z}$.

3. (12 points) Let $f = x^2 + x + 1$ and let α denote the residue of x in the ring $R = \mathbb{Z}[x]/(f)$. Express $(\alpha^2 + 1)^2$ in terms of the basis $(1, \alpha)$ of R .

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4. (12 points) Determine the division with remainder in $\mathbb{Z}[i]$. Let $\sigma(a) = |a|^2$ be the size function on $\mathbb{Z}[i]$. Find $q, r \in \mathbb{Z}[i]$ such that $6 = (1 + 2i)q + r$ and $\sigma(r) < \sigma(1 + 2i)$.

5. (12 points) Let $\mathbb{Z}[\sqrt{-3}] = \{m + n\sqrt{-3} \mid m, n \in \mathbb{Z}\}$. Is $(\sqrt{-3} + 2)$ a maximal ideal in $\mathbb{Z}[\sqrt{-3}]$? Why?

6. (12 points) What are the maximal ideals of $\mathbb{C}[x, y]/(x^2 - 1, y^2)$?

7. (12 points) Find the kernel of the homomorphism $\varphi: \mathbb{C}[x, y] \rightarrow \mathbb{C}[t]$ determined by $\phi(x) = t + 1$, $\phi(y) = t^2$ and $\phi(c) = c$ for any $c \in \mathbb{C}$.

8. (12 points) Give an example of irreducible polynomial $f(x)$ of degree 3 in $\mathbb{F}_2[x]$.

9. (12 points) Prove that $\mathbb{Z}[i]/(5)$ is not a field.