s.t. $\bar{f}\,\pi = f$.

$$\begin{tikzcd} R \arrow[r, "f"] \arrow[d, "\pi"'] & R' \\ R/I \arrow[ur, dashed, "\bar{f}"'] & \end{tikzcd}$$

b) If $K = I$, $\bar{f}$ is isomorphism
and f surjective.

Thm (Correspondence Thm)

$\varphi: R \to R'$ is surjective ring homomorphism. $K := \ker\varphi$

$\{$ ideals in R containing $K\}$

$\longleftrightarrow$ $\{$ideals in $R'\}$

- If $I \supset K$, then $\varphi(I)$ is an ideal in R'.
- If $\bar{I}$ is an ideal in R', then

$\varphi^{-1}(\bar{I})$ is an ideal in R.

step 1. $\varphi(I)$ is an ideal in R'.

$\varphi^{-1}(\bar{I})$ is an ideal in R

step 2: $\varphi(\varphi^{-1}(\bar{I})) = I$. $\quad \varphi^{-1}(\varphi(I)) = I$

Ex: $\varphi: \mathbb{C}[x, y] \to \mathbb{C}[t]$.

$x \mapsto t$.

$y \mapsto t^2$.

$\ker\varphi = (y - x^2)$.

why? $\quad g(x, y) \mapsto 0$,

$g(t, t^2) = 0$.

$f = y - x^2 \in \mathbb{C}[x][y]$

$g = f \cdot q + r \qquad r \in \mathbb{C}[x][y]$

deg in $y < 1$.

so $r(x,y) = r(x)$, and $r(t,t^2) = r(t) = 0$

and $r(t) = 0 \Rightarrow r = 0$.

so $g = f \cdot q$.

Ideals containing $(y - x^2)$

$\longleftrightarrow$ ideals in $\mathbb{C}[t]$.

$(f(t))$.

$\varphi^{-1}(f(t)) = (f(x), y - x^2)$.

Ex: $\mathbb{C}[t]/(t^2 - 1) = R'$

f monic.

any ideal in R' is (f). f divides $t^2 - 1$.

If $\deg f = 0$. $f = 1$.

$\deg f = 1$. $f = t - \alpha$.

$t-\alpha$ divides $h(t)$ means

$h(\alpha)=0$. (Use division with remainder)

So $f=(t-1)$. or $(t+1)$.

If $\deg f=2$. $f=t^2-1$.

Useful facts: $I=(a)$.

$J=(b)$

$I\subset J$ iff b divides a.

Adjoining elements.

$$R/(a,b) \cong \left. R/(a) \middle/ (\bar{b}) \right.$$

Ex: $\left(\mathbb{Z}[i]/i-2 \right)$

$\mathbb{Z}[i]$ is the image of

$\mathbb{Z}[x] \to \mathbb{C}$.

$x \mapsto i$.

$$\mathbb{Z}[i] \cong \mathbb{Z}[x]/(x^2-1).$$

why : $\varphi : \mathbb{Z}[x] \to \mathbb{C}$.
$x \mapsto i$.

$\ker \varphi = (x^2+1)$,

if $g(x) \in \ker \varphi$

then $g(i) = 0$, $g(-i) = 0$.

so $g(x) = (x^2+1) \cdot q(x) + r(x)$

$\deg r \leq 1$, but $i \notin \mathbb{Z}$,

so $r(x) = 0$.

$$\left. \mathbb{Z}[x]/(x^2+1) \right/ x-2 \cong \frac{\mathbb{Z}[x]/(x-2)}{(x^2+1)}$$

$$\cong \mathbb{Z}/(5)$$

Here we use $\mathbb{Z}[x]/(x-2) \cong \mathbb{Z}$.
$x \mapsto 2$.