Review:

Defns: Rings, units, ideals. principal ideals.

polynomial ring $R[x]$. quotient ring.

homomorphism, subring, kernel.

Product ring, maximal ideals.

idempotent element. characteristic

integral domain, divisor. prime element, zero divisor

irreducible element, associates.

Euclidean ring $\Rightarrow$ PID $\Rightarrow$ UFD

Fractions.

Thms: ① substitution principle, extend $\varphi: R \to R'$ (homomorphism)

to $R[x] \to R'$.

$x \mapsto a$

② correspondence thm

Ex: Find ideals containing $(y-x^2)$ in $\mathbb{C}[x,y]$

$\varphi: \mathbb{C}[x,y] \to \mathbb{C}[t]$.

$x \mapsto t$

$y \mapsto t^2$.

$\ker\varphi = (y-x^2)$

Find maximal ideals in $\mathbb{Z}/8\mathbb{Z}$.

③ Adding relations.

$$R/(a,b) \cong R/(a) \Big/ (b)$$

$$\cong R/(b) \Big/ (a)$$

$$\mathbb{Z}[i]/(i+3)$$

④ Adjoining elements.

$R[x]/f(x)$. $\quad f(x) = x^n + a_{n-1}x^{n-1} + \dots + 1$.

then $R[x]/(f(x))$ has a basis

$1, \alpha, \dots \alpha^{n-1}$

⑤ Division with remainder.

5.1 How to calculate in $\mathbb{F}[x]$

5.2. How to calculate in $\mathbb{Z}[i]$

(6) Euclidean domain $\Rightarrow$ PID $\Rightarrow$ UFD

$\mathbb{Z}[i]$ is Euclidean domain

(7) Hilbert's Nullstellensatz.

(8) Maximal ideals in $\mathbb{Z}$, and $\mathbb{F}[x]$ ($\mathbb{F}$ a field)

(9) I is maximal ideal iff R/I is a field.

(10) In PID, prime $\Leftrightarrow$ irreducible.

Useful techniques.

① change of variable in $\mathbb{C}[t]$, or

$\mathbb{C}[x,y]$

② If R is an integral domain.

In $R[x]$. $\deg f(x) + \deg g(x)$

$= \deg(f(x) \cdot g(x))$