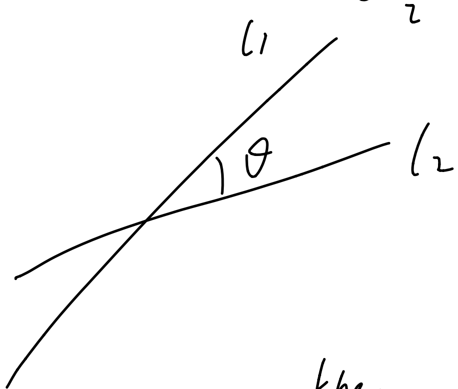
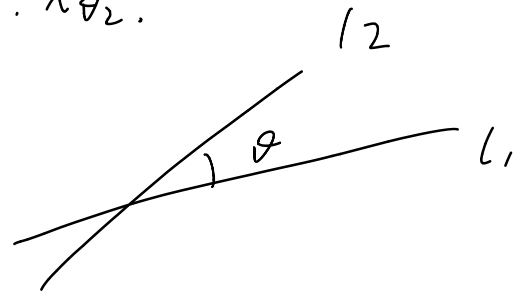
1. Assume $y_1 = y_{\frac{\theta_1}{2}} = y_0 \cdot x_{\theta_1}$.

$$y_2 = y_{\frac{\theta_2}{2}} = y_0 \cdot x_{\theta_2}.$$





then $\theta = \pm\left(\frac{1}{2}\theta_1 - \frac{1}{2}\theta_2\right)$

$$y_1 y_2 = y_0 \cdot x_{\theta_1}\, y_0\, x_{\theta_2}$$

$$= x_{-\theta_1}\, x_{\theta_2} = x_{\theta_2-\theta_1}$$

So $y_1 y_2 = x_{2\theta}$ or $x_{-2\theta}$

$$= \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \text{ or } \begin{bmatrix} \cos 2\theta, & \sin 2\theta \\ -\sin 2\theta, & \cos 2\theta \end{bmatrix}$$

2. $f: SO(2) \longrightarrow SO(2)$

$$x_\theta \longmapsto x_{3\theta}$$

① f well-defined because

$X_{\theta_1} = X_{\theta_2}$ iff $\theta_1 - \theta_2 = 2k\pi$, $k \in \mathbb{Z}$,

then $X_{3\theta_1} = X_{3\theta_2}$ if $X_{\theta_1} = X_{\theta_2}$

② $f(X_{\theta_1} \cdot X_{\theta_2}) = f(X_{\theta_1+\theta_2})$

$= X_{3(\theta_1+\theta_2)}$

$= f(X_{\theta_1}) \cdot f(X_{\theta_2})$

So f is a group homomorphism.

③ $\ker f = \{ X_\theta \mid X_{3\theta} = X_0 \}$.

$= \{ X_\theta \mid 3\theta \equiv 0 \mod 2\pi \}$

$= \{ X_{\frac{2\pi}{3}}, X_{-\frac{2\pi}{3}}, X_0 \}$.