5, c) $SL(2,\mathbb{C})$ operation preserves $\langle\, ,\, \rangle$

$$\langle PA, PA' \rangle = \det(P(A+A')P^*) - \det(PAP^*) - \det(PA'P^*)$$

$$= (\det P) \det(A+A') \det(P^*) - (\det P) \det A \det P^* - (\det P) \det A' \det P^*$$

$$= \det(A+A') - \det A - \det A'$$

$$= \langle A, A' \rangle$$

So the operation induces a group homomorphism $\varphi: SL(2,\mathbb{C}) \to O_{1,3}$.

If $p \in \ker \varphi$

$$p \cdot A = A \quad \forall A \in W$$

$$PAP^{-1} = A \quad \forall A \in W.$$

Use $A = \begin{bmatrix} 1 & \\ & -1 \end{bmatrix}$

$$P \begin{bmatrix} 1 & \\ & -1 \end{bmatrix} = \begin{bmatrix} 1 & \\ & -1 \end{bmatrix} P$$

$P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\Downarrow$ $b = c = 0$

so $P = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$.

Use $A = \begin{bmatrix} & 1 \\ 1 & \end{bmatrix}$

$PA = AP \Rightarrow a = d$.

so $P = a\begin{bmatrix} 1 & \\ & 1 \end{bmatrix}$.

since $\det P = 1$, $a = \pm 1$.

$\ker \varphi \subset \{\pm I\}$.

$\{\pm I\} \subset \ker \varphi$ because

$(\pm I)A = A(\pm I) \quad \forall A \in W$.