Math 371 Homework#3

Due on 2/13 at the beginning of Lecture

1. Artin, Chapter 6, problem 7.7.
2. Artin, Chapter 6, problem 7.8.
3. Artin, Chapter 9, problem 3.1.
4. Artin, Chapter 9, problem 4.3.
5. Artin, Chapter 9, problem 4.8. Here Hermitian matrix means complex square matrix A such that $A^* = A$.
6. Let W be the space of real skew-symmetric 3×3 matrices, i.e. $W = \{A \in M_{3\times 3}(\mathbb{R}) | A = -A^T\}$. Prove that $P * A = PAP^t$ defines an operation of SO_3 on W. Try to find a positive definite symmetric bilinear form on W which is invariant under this operation.
7. Let S_3 be the permutation group of three elements $\{1,\ 2,\ 3\}$. Denote by $e_1 = (1,0,0)^T, e_2 = (0,1,0)^T, e_3 = (0,0,1)^T$ the standard basis of $\mathbb{C}^3$. Define a linear operation of S_3 on $\mathbb{C}^3$ by $\sigma e_i = e_{\sigma(i)}$. What is $\sigma(\sum_i a_i e_i)$? Write down the matrix representation R under the standard basis e_1, e_2, e_3 and compute the character χ_R