Math 371 Homework#6

Due on 3/19 at the beginning of Lecture

1. Prove the intersection of kernels of 1-dimensional irreducible characters gives the commutator subgroup $G' = [G, G]$ of G. You can use the following universal property of commutator group. A normal subgroup N of G induces an abelian quotient group G/N if and only if N contains G'.

2. From class, we know that the character χ_{reg} of regular representation satisfies $\chi_{reg}(g) = 0$ if $g \neq e$. There is an inverse of this proposition. Let χ be a character of G and satisfies $\chi(g) = 0$ if $g \neq e$. Prove that the corresponding representation is the direct sum of several copies of regular representation, i.e. $\rho \cong n\rho_{reg}$ for some integer n.

3. Find the character table of dihedral group D_4. Here D_4 is the symmetry group of a square and is generated by x the rotation by $\pi/2$ and y a reflection. From calculation of $O(2)$, we know $x^4 = e, y^2 = e, yx = x^{-1}y$.

4. Let χ be a faithful character of G, i.e. $\ker \chi = \{e\}$. Prove that G is abelian group if and only if all the irreducible components appearing in the irreducible decomposition of χ are 1-dimensional. (Hint: use question 1)

5. Artin chapter 10, 7.4

6. Let G operate on a finite set S and ρ be the induced permutation representation. Prove that the multiplicity of trivial representation ρ_1 appearing in irreducible representation decomposition of ρ is equal to the number of orbits of this operation.