Math 371 Homework#8

Due on 4/14

1. **Artin Chapter 11, 5.1** Let $f = x^4 + x^3 + x^2 + x + 1$ and let α denote the residue of x in the ring $R = \mathbb{Z}[x]/(f)$. Express $(\alpha^3 + \alpha^2 + \alpha)(\alpha^5 + 1)$ in terms of the basis $(1, \alpha, \alpha^2, \alpha^3)$ of R.

2. Use the Euclidean domain structure described in Proposition 12.2.5 to divide -4 by $2+i$ in $\mathbb{Z}[i]$, i.e. find $q, r \in \mathbb{Z}[i]$ such that $-4 = (2+i)q+r$ and $r = 0$ or $\sigma(r) < \sigma(2+i)$. (Remark: the textbook uses a geometric proof and we did a more computational proof in class)

3. Assume a and b are associates in integral domain R. Prove that if a is irreducible, then b is also irreducible.

4. Is $(i - 2)$ a maximal ideal in the ring of Gaussian integers $\mathbb{Z}[i]$? Why? (Hint: use the previous homework of how to identify the quotient ring with $\mathbb{Z}/n\mathbb{Z}$)

5. Is $(i + 3)$ a maximal ideal in the ring of Gaussian integers $\mathbb{Z}[i]$? Why?

6. Prove that the ring $\mathbb{Z}[\sqrt{-2}]$ is a Euclidean domain, hence PID. (Hint: use the same idea in the proof for $\mathbb{Z}[\sqrt{-1}]$)

7. Prove that $\mathbb{C}[x, y]$ is not a PID (principal ideal domain). (Hint: consider the ideal (x, y) and prove that it can not be generated by one element. Assume it is generated by one element $f(x, y)$, then try to find the degrees of $f(x, y)$ with respect to x and y if x and y are multiples of $f(x, y)$.)