

代数 1 H 班 作业 9

2022 年 11 月 16 日

题 1. Artin, Chapter 14, 1.4

题 2. Artin, Chapter 14, 2.1

题 3. Artin, Chapter 14, 2.2

题 4. Artin, Chapter 14, 2.4

题 5. Assume Cayley-Hamilton theorem holds for any fields, prove Cayley-Hamilton theorem for any commutative ring R . In other words, let matrix $A \in M_n(R)$, and characteristic polynomial $f(\lambda) = \det(\lambda I - M) = \lambda^n + s_{n-1}\lambda^{n-1} \cdots + s_0 \in R[\lambda]$. Prove that $f(A) = 0$.

题 6. The coefficients of characteristic polynomial can define functions of $s_i: M_n(R) \rightarrow R$. Prove that s_i satisfies $s_i(AB) = s_i(BA)$.

题 7. Artin, Chapter 14, 6.1

题 8. Artin, Chapter 14, 6.2

题 9. Prove that \mathbb{Q}/\mathbb{Z} is not a finitely generated \mathbb{Z} -module. How about $\mathbb{C}(t)/\mathbb{C}[t]$ as a $\mathbb{C}[t]$ module?

题 10. Let M be a R -module and $N \subset M$ be a submodule. Prove that $M \cong N \oplus (M/N)$ if M/N is a free R -module.

题 11. Give an example of non-noetherian ring.

题 12. Let R be the ring of algebraic integers in imaginary quadratic extension of \mathbb{Q} . If a R -module is generated by n -elements, how many generators are needed to generate its submodule? Try to give an estimate on the minimal number of generators needed.

题 13. *Artin, Chapter 14, M.8*

题 14. *We call a module cyclic if it is generated by one element. Classify cyclic $\mathbb{R}[t]$ -modules.*