(Sylow Thm)

Classify G with order 6.

$K = \langle b \rangle$

K Sylow 3-subgroup K normal.

H Sylow 2-subgroup, $H = \langle a \rangle$

$H \cap K = \{e\}$. $\Rightarrow$ claim:

$K \times H \to G$ injective

$\Rightarrow$ $K \times H \to G$ bijective.

$b^i a^j b^k a^l$

$= b^i \underbrace{(a^j b^k a^{-j})}_{\in K \text{ (because } K \text{ normal)}} a^{j+l}$

so $b^i \underbrace{(a^j b^k a^{-j})}_{\text{what are the possible choice}} \cdot a^{j+l}$

$aba^{-1} = b^m$.

Then $a^2 b a^{-2} = a(aba^{-1})a^{-1}$

$= a b^m a^{-1} = (b^m)^m = b^{m^2}$

$\Rightarrow m^2 \equiv 1 \pmod 3$

$\Rightarrow m = 1$, or, -1

$\Rightarrow aba^{-1} = b^{-1} \Rightarrow G \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$

$aba^{-1} = b^{-1} \Rightarrow G \cong D_3$ or S_3

Setting: $H, K \subseteq G$ subgroups. K normal

$KH = G$, $K \cap H = \{e\}$

Then $K \times H \to G$ bijective.

$\rho: H \longrightarrow \mathrm{Aut}(K)$ <u>verify group homo.</u>

$h \longmapsto (k \mapsto hkh^{-1})$

then $(k_1, h_1) \cdot (k_2 h_2)$

$= \underline{k_1 \, \rho(h_1)(k_2)} \, h_1 h_2$

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semi-direct product:

H, K any two groups

$\rho : H \to \mathrm{Aut}(K)$

Define binary operation on $H \times K$

by $(h_1, k_1)(h_2, k_2)$

$= (h_1 h_2, k_1 \rho(h_1) k_2)$

<u>Verify: this is a group</u>

Useful facts: $\mathrm{Aut}(\mathbb{Z}/n\mathbb{Z}) \cong (\mathbb{Z}/n\mathbb{Z})^\times$

explicit form: $G = \langle a \rangle$

$\rho : G \to G$ iso $\Rightarrow$ $\mathrm{ord}(a), m$

$a \mapsto a^m$, coprime.

Try: $\underline{\# G = 21}$

$\# G = 12$, $\# G = 18$.

Semi-direct product.

$D_n = \langle a, b \rangle$.

a rotation, b reflection

$bab^{-1} = a^{-1}$,
